11-2012

Proving Operations with Numbers by Using Manipulatives

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Plan for the hour-long session

- Explore mathematics concepts related to operations with numbers (10 minutes)
- Create developmentally-appropriate proofs with manipulatives (40 minutes)
- Discussion focusing on instructional implications and opportunity to ask questions (10 minutes)

Context for this session

- Young students are likely to make sense of mathematics when they use concrete materials before engaging in abstract symbol manipulation (Bruner, 1985).

- Young students can reason about mathematical theorems and as they mature, offer developmentally-appropriate proofs (NCTM, 2000).

- CCSSM indicates that mathematically proficient students should “construct viable arguments and critique the reasoning of others” (i.e., SMP #3) as well as “look for and make use of structure (i.e., SMP #7) (CCSSO, 2010, pp. 6-8).

Further context

- Representing a proof and communicating the proof (Harel & Sowder, 2007).
- Teachers should be prepared to scaffold K-12 students while engaging in reasoning and proof (Knuth, 2002; Reid & Zack, 2009).
- Thus, preservice teachers (PSTs) need proof and reasoning-related activities that support examining various representations of proofs and communicating one's reasoning and/or proof.

Mathematics Norms

- We will always look for another approach to solve problems.
- We will use multiple representations (e.g., pictures, graphs, tables, symbols, numbers, manipulatives, and words) to assist us while doing mathematics.
- We will persist with every problem and examine it from multiple perspectives.
- We will be mathematically precise whenever possible.
- We will justify our mathematics work whenever possible.

Sociomathematical Norms

- If we disagree with someone or are unclear about their ideas related to mathematics content and pedagogy, we will ask a question about his or her idea and describe why we disagree or are confused.
- We will ask questions when we do not understand something about mathematics content and pedagogy.
- We will comment on others' ideas about mathematics content and pedagogy rather than the person.
- When presenting work, we will ask whether there are questions and then applaud the presenter for sharing his/her ideas.

What ideas are in the CCSSM for grades 3-5?

- Multiplication and division with whole numbers. (3.OA.3)
- Understand the relationship between multiplication and addition. (3.OA.5)
- Recognize factors and multiples up to 100. (4.OA.4)
- Apply knowledge of multiplication and division to fractions. (5.NF.4)

CCSSM and Proofs to Ponder

- Multiplication and division with whole numbers (3.OA.3)
- Understand the relationship between multiplication and addition (3.OA.5)
- Recognize factors and multiples up to 100 (4.OA.4)
- Apply knowledge of multiplication and division to fractions (5.NF.4)
- How can we prove that multiplication and division undo each other for whole numbers? (third grade)
- How can we prove the divisibility rule for two? (fourth grade)
- How can we prove that multiplication and division undo each other for rational numbers greater than 0? (fifth grade)

Prove it!

- How can we prove that multiplication and division undo each other for whole numbers? (third grade)
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K-12 Instructional Implications

- When do I put proof into my CCSSM-aligned instruction for K-12 students?
  - When discussing mathematics concepts and procedures AND it's developmentally appropriate.
- When will I have time for this?
  - Reasoning and sense making are central to the new standards. Make time.
- Why is this important?
  - Proof and reasoning are process standards that students need to experience and demonstrate.
- Isn't reasoning and proof too advanced for 8-12 year olds?
  - No, any student can and should engage in reasoning and proof.

Instructional Implications for PSTs

- When do I put proof into my instruction for PSTs?

- How might their perceptions of proof and reasoning impact
  - their engagement in the proof-related activity?
  - their representations used while engaging in reasoning and proof-related activities?
  - classroom discourse while engaging in reasoning and proof-related activities?

Thanks for coming!

Do you have any questions?

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